

SUM RULES FOR TOTAL CROSS-SECTIONS OF HADRON PHOTO-PRODUCTION ON PSEUDOSCALAR MESONS AND OCTET BARYONS

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Considering very high energy peripheral electron-hadron scattering with a production of hadronic state X moving closely to the direction of initial hadron the Weizsäcker-Williams like expression, relating the difference of q^2 -dependent differential cross-sections of DIS processes to the convergent integral over the difference of the total hadron photo-production cross-sections on hadrons, is derived. Then, exploiting analytic properties of the forward retarded Compton scattering amplitude on the same hadrons, first, the sum rules are derived bringing into relation hadron electromagnetic form factors with the difference of the q^2 -dependent differential cross-sections of DIS, then using Weizsäcker-Williams like expression and taking the derivative of both sides according to \mathbf{q}^2 for $\mathbf{q}^2 \rightarrow 0$ one comes to new universal hadron sum rules relating hadron static parameters to the convergent integral over the difference of the total hadron photo-production cross-sections on hadrons.

I. WEIZSÄCKER-WILLIAMS LIKE RELATIONS

In a derivation of them one considers a very high energy peripheral electro-production process on hadrons h

$$e^-(p_1) + h(p) \rightarrow e^-(p'_1) + X, \quad (1)$$

with the produced pure hadronic state X moving closely to the direction of the initial hadron to be described by the matrix element

$$M = i \frac{\sqrt{4\pi\alpha}}{q^2} \bar{u}(p'_1) \gamma_\mu u(p_1) \langle X | J_\nu^{EM} | h \rangle g^{\mu\nu}, \quad (2)$$

in the one photon exchange approximation with $m_X^2 = (p + q)^2$.

Now, applying the Sudakov expansion of the photon transferred four-vector q

$$q = \beta_q \tilde{p}_1 + \alpha_q \tilde{p} + q^\perp, \quad q_\perp = (0, 0, \mathbf{q}), \quad q_\perp^2 = -\mathbf{q}^2 \quad (3)$$

into the almost light-like vectors

$$\tilde{p}_1 = p_1 - m_e^2 p / (2p_1 p), \quad \tilde{p} = p - m_B^2 p_1 / (2p_1 p) \quad (4)$$

and also the Gribov prescription for the numerator of the photon Green function

$$g_{\mu\nu} = g_{\mu\nu}^\perp + \frac{2}{s}(\tilde{p}_\mu \tilde{p}_{1\nu} + \tilde{p}_\nu \tilde{p}_{1\mu}) \approx \frac{2}{s} \tilde{p}_\mu \tilde{p}_{1\nu} \quad (5)$$

with $s = (p_1 + p)^2 \approx 2p_1 p \gg Q^2 = -q^2$ in (2), then for very high electron energy in (1) and small photon momentum transfer squared $t = q^2 = -Q^2 = -\mathbf{q}^2$ the cross-section can be written in the form

$$d\sigma^{e^-h \rightarrow e^-X} = \frac{4\pi\alpha}{s(q^2)^2} p_1^\mu p_1^\nu \times \quad (6)$$

$$\sum_{X \neq h} \sum_{r=-j}^j \langle h^{(r)} | J_\mu^{EM} | X \rangle^* \langle X | J_\nu^{EM} | h^{(r)} \rangle d\Gamma$$

with a summation through the created hadronic states X and the spin states of the initial hadron.

If the relation $\int d^4q \delta^{(4)}(p_1 - q - p'_1) = 1$ is used in the phase space volume $d\Gamma$ of the final particles, then

$$d\Gamma = \frac{ds_1}{2s(2\pi)^3} d^2\mathbf{q} d\Gamma_X \quad (7)$$

with

$$d\Gamma_X = (2\pi)^4 \delta^{(4)}(p + q - \sum_i^n q_i) \prod_i^n \frac{d^3q_i}{2E_i(2\pi)^3} \quad (8)$$

$$s_1 = 2(qp) = m_X^2 + \mathbf{q}^2 - m_h^2 = s\beta_q. \quad (9)$$

Moreover, the current conservation condition ($\alpha_q \tilde{p}$ gives a negligible contribution)

$$q^\mu \langle X | J_\mu^{EM} | h^{(r)} \rangle \approx \quad (10)$$

$$\approx (\beta_q \tilde{p}_1 + q_\perp)^\mu \langle X | J_\mu^{EM} | h^{(r)} \rangle = 0,$$

is applied in order to utilize in the expression for cross-section the relation

$$\int p_1^\mu p_1^\nu \sum_{X \neq h} \sum_{r=-j}^j \langle h^{(r)} | J_\mu^{EM} | X \rangle^* \quad (11)$$

$$\langle X | J_\nu^{EM} | h^{(r)} \rangle d\Gamma_X = 2i \frac{s^2}{s_1^2} \mathbf{q}^2 \text{Im} \tilde{A}^{(h)}(s_1, \mathbf{q}),$$

with the imaginary part of the retarded forward Compton scattering amplitude $\tilde{A}^h(s_1, \mathbf{q})$ on a hadron.

As a result for a difference of corresponding differential cross-sections of the electro-production on h and h' (after integration in the cross-section (6) over $d\Gamma_X$, as well as over the invariant mass squared m_X^2 , i.e. over the variable s_1 to be interested only for \mathbf{q} distribution) one finds the expression

$$\begin{aligned} & \left(\frac{d\sigma^{e^-h \rightarrow e^-X}(s, \mathbf{q})}{d^2\mathbf{q}} - \frac{d\sigma^{e^-h' \rightarrow e^-X'}(s, \mathbf{q})}{d^2\mathbf{q}} \right) = \\ &= \frac{\alpha \mathbf{q}^2}{4\pi^2} \int_{s_1^{thr}}^{\infty} \frac{ds_1}{s_1^2 [\mathbf{q}^2 + (m_e s_1/s)^2]^2} \times \\ & \times [Im\tilde{A}^h(s_1, \mathbf{q}) - Im\tilde{A}^{h'}(s_1, \mathbf{q})]. \end{aligned} \quad (12)$$

Finally, if one neglects the second term in square brackets of the denominator of the integral in (12) (as m_e is small and s is large in comparison with s_1), the expressions $d^2\mathbf{q} = \pi d\mathbf{q}^2$ is exploited, the limit $\mathbf{q}^2 \rightarrow 0$ along with the relation $Im\tilde{A}^h(s_1, \mathbf{q}) = 4s_1 \sigma_{tot}^{\gamma^*h \rightarrow X}(s_1, \mathbf{q})$ is applied, one comes to Weizsäcker-Williams like expressions

$$\begin{aligned} & \mathbf{q}^2 \left(\frac{d\sigma^{e^-h \rightarrow e^-X}}{d\mathbf{q}^2} - \frac{d\sigma^{e^-h' \rightarrow e^-X'}}{d\mathbf{q}^2} \right)_{|\mathbf{q}^2 \rightarrow 0} = \\ &= \frac{\alpha}{\pi} \int_{s_1^{thr}}^{\infty} \frac{ds_1}{s_1} [\sigma_{tot}^{h \rightarrow X}(s_1) - \sigma_{tot}^{h' \rightarrow X}(s_1)], \end{aligned} \quad (13)$$

relating the difference of q^2 -dependent differential cross-sections of the DIS processes to the convergent integral over the difference of the total hadron photo-production cross-sections on hadrons.

From the relation (13) one can see immediately that considering differences of the total cross-sections one achieves the convergent integral.

II. VIRTUAL COMPTON SCATTERING ON HADRON

In the previous section we have used the concept "the retarded forward Compton scattering amplitude on hadron". Here we slightly clarify it.

The total virtual photon Compton scattering amplitude $A(s_1, \mathbf{q})$ consists of the four different contributions to be classified according to the corresponding Feynman diagrams

$$A(s_1, \mathbf{q}) = \tilde{A}(s_1, \mathbf{q}) + A_a(s_1, \mathbf{q}) + A_P(s_1, \mathbf{q}) + A_{odd}. \quad (14)$$

$\tilde{A}(s_1, \mathbf{q})$ represents a class of diagrams in which the initial state photon is first absorbed by a hadron line and then emitted by the scattered hadron - retarded Compton scattering amplitude

$A_a(s_1, \mathbf{q})$ represents a class of diagrams in which the scattered photon is first emitted along the hadron line and the point of absorption is located later on - advanced Compton scattering amplitude

$A_P(s_1, \mathbf{q})$ represents a class of diagrams in which both photons do not interact with the initial hadron line, in other words it corresponds to the Pomeron - type Feynman diagrams and gives the non-vanishing contributions to the total cross-sections in the limit of a large invariant mass squared of initial particles $s_1 \rightarrow \infty$

A_{odd} can be relevant in experiments measuring charge-odd effects

For more detail see Ref.[1]

III. q^2 DEPENDENT MESON AND BARYON SUM RULES

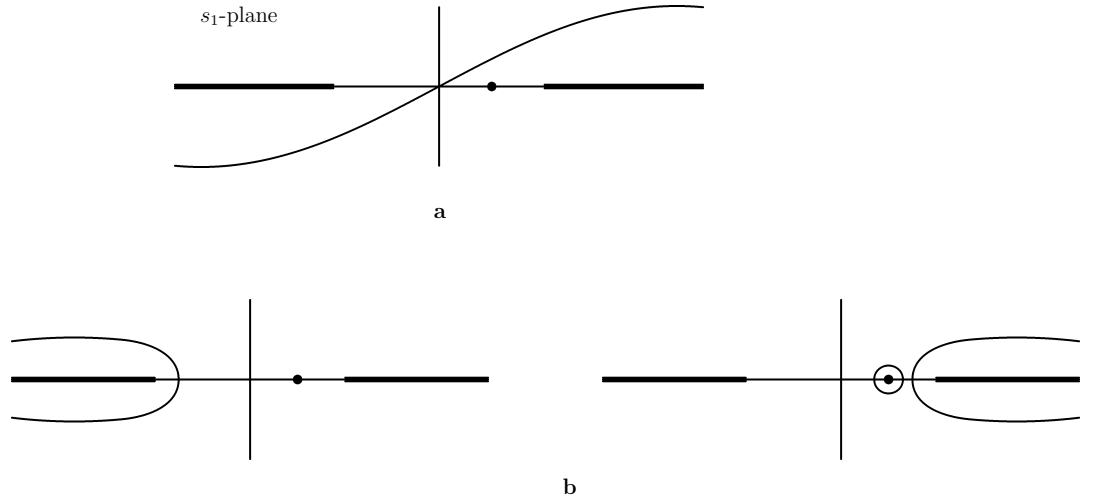


FIG. 1: Sum rule interpretation in s_1 plane.

Universal sum rules are derived by investigating the analytic properties of the retarded Compton scattering amplitude $\tilde{A}^h(s_1, \mathbf{q})$ in s_1 - plane as presented in Fig. 1a, then defining the integral I over the path C (for more detail see [2]) in the s_1 -plane

$$I = \int_C ds_1 \frac{p_1^\mu p_1^\nu}{s^2} \left(\tilde{A}_{\mu\nu}^h(s_1, \mathbf{q}) - \tilde{A}_{\mu\nu}^{h'}(s_1, \mathbf{q}) \right) \quad (15)$$

from the gauge invariant light-cone projection $p_1^\mu p_1^\nu \tilde{A}_{\mu\nu}^h(s_1, \mathbf{q})$ of the amplitude $\tilde{A}^h(s_1, \mathbf{q})$ and once closing the contour C to upper half-plane, another one to lower half-plane (see Fig. 1b).

As a result the following sum rule appears

$$\pi(Res^{h'} - Res^h) = \mathbf{q}^2 \int_{r.h.}^{\infty} \frac{ds_1}{s_1^2} [Im\tilde{A}^h(s_1, \mathbf{q}) - Im\tilde{A}^{h'}(s_1, \mathbf{q})]. \quad (16)$$

The left-hand cut contributions expressed by an integral over the difference $[Im\tilde{A}^h(s_1, \mathbf{q}) - Im\tilde{A}^{h'}(s_1, \mathbf{q})]$ are assumed to be mutually annulated.

Now, if the meson or baryon sum rules are considered, one has to take into account the corresponding residuum of the intermediate state pole (see Fig. 1).

If one considers mesons - their electromagnetic structure is described by one charge form factor and the residuum takes the form

$$Res^{(M)} = 2\pi\alpha F_M^2(-\mathbf{q}^2). \quad (17)$$

If one considers baryons - their electromagnetic structure is described by Dirac and Pauli form factors and the residuum takes different form

$$Res^B = 2\pi\alpha(F_{1B}^2 + \frac{\mathbf{q}^2}{4m_B^2}F_{2B}^2), \quad (18)$$

where in both cases an averaging over the initial hadron and photon spins is performed.

Then, substituting (17) and (18) into (16) and taking into account (12) from the previous Section with $d^2\mathbf{q} = \pi d\mathbf{q}^2$, one comes to the q^2 -dependent meson sum rule [3]

$$\begin{aligned} & [F_{P'}^2(-\mathbf{q}^2) - F_{P'}^2(0)] - [F_P^2(-\mathbf{q}^2) - F_P^2(0)] = \\ & = \frac{2}{\pi\alpha^2}(\mathbf{q}^2)^2 \left(\frac{d\sigma^{e^-P \rightarrow e^-X}}{d\mathbf{q}^2} - \frac{d\sigma^{e^-P' \rightarrow e^-X'}}{d\mathbf{q}^2} \right), \end{aligned} \quad (19)$$

and the q^2 -dependent baryon sum rule [4]

$$\begin{aligned} & [F_{1B'}^2(-\mathbf{q}^2) - F_{1B'}^2(0)] - [F_{1B}^2(-\mathbf{q}^2) - F_{1B}^2(0)] + \\ & + \mathbf{q}^2 \left[\frac{F_{2B'}^2(-\mathbf{q}^2)}{4m_{B'}^2} - \frac{F_{2B}^2(-\mathbf{q}^2)}{4m_B^2} \right] = \\ & = \frac{2}{\pi\alpha^2}(\mathbf{q}^2)^2 \left(\frac{d\sigma^{e^-B \rightarrow e^-X}}{d\mathbf{q}^2} - \frac{d\sigma^{e^-B' \rightarrow e^-X}}{d\mathbf{q}^2} \right), \end{aligned} \quad (20)$$

respectively, where the left-hand side in both cases was re-normalized in order to separate the pure strong interactions from electromagnetic ones.

IV. UNIVERSAL SUM RULE FOR TOTAL HADRON PHOTO-PRODUCTION CROSS-SECTIONS ON MESONS

Now, employing the Weicsäcker-Williams like relation for mesons, taking a derivative according to \mathbf{q}^2 of both sides in q^2 -dependent meson sum rule for $\mathbf{q}^2 \rightarrow 0$ and using the laboratory reference frame by $s_1 = 2m_B\omega$, one comes to the new universal meson sum rule [3] relating meson mean square radii to the integral over a difference of the corresponding total photo-production cross-sections on mesons

$$\begin{aligned} & \frac{1}{3}(\langle r_{P'}^2 \rangle - \langle r_P^2 \rangle) = \\ & = \frac{2}{\pi^2\alpha} \int_{\omega_P}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma P \rightarrow X}(\omega) - \sigma_{tot}^{\gamma P' \rightarrow X}(\omega)], \end{aligned} \quad (21)$$

in which just a mutual cancellation of the rise of the latter cross sections for $\omega \rightarrow \infty$ is achieved.

V. APPLICATION TO VARIOUS COUPLES OF MESONS

According to the SU(3) classification of existing hadrons the following ground state pseudoscalar meson nonet $\pi^-, \pi^0, \pi^+, K^-, \bar{K}^0, K^0, K^+, \eta, \eta'$ exists. However, in consequence of CPT invariance the meson electromagnetic form factors $F_P(-\mathbf{q}^2)$ hold the following relation

$$F_P(-\mathbf{q}^2) = -F_{\bar{P}}(-\mathbf{q}^2), \quad (22)$$

where \bar{P} means antiparticle.

Since π_0, η and η' are true neutral particles, their electromagnetic form factors are owing to the (22) zero in the whole region of a definition and therefore we exclude them from further considerations.

If one considers couples of particle-antiparticle like π^\pm, K^\pm and K^0, \bar{K}^0 , the left hand side of (19) is owing to the relation (22) equal zero and we exclude couples π^\pm, K^\pm and K^0, \bar{K}^0 from further considerations as well.

If one considers a couple of the iso-doublet of kaons K^+, K^0 and K^-, \bar{K}^0 , the following Cabibbo-Radicati like sum rules [5] for kaons can be written

$$\frac{1}{6}\pi^2\alpha\langle r_{K^+}^2 \rangle = \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma K^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma K^0 \rightarrow X}(\omega)] \quad (23)$$

$$\frac{1}{6}\pi^2\alpha(-1)\langle r_{K^-}^2 \rangle = \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma K^- \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \bar{K}^0 \rightarrow X}(\omega)], \quad (24)$$

in which the relation $\langle r_{K^+}^2 \rangle = -\langle r_{K^-}^2 \rangle$ for kaon mean squared charge radii, following directly from (22), holds and divergence of the integrals, due to an increase of the total cross-sections $\sigma_{tot}^{\gamma K^\pm \rightarrow X}(\omega)$ for large values of ω , is taken off by the increase of total cross-sections $\sigma_{tot}^{\gamma K^0 \rightarrow X}(\omega)$ and $\sigma_{tot}^{\gamma \bar{K}^0 \rightarrow X}(\omega)$, respectively. If besides the latter, also the relations

$$\begin{aligned}\sigma_{tot}^{\gamma K^0 \rightarrow X}(\omega) &\equiv \sigma_{tot}^{\gamma \bar{K}^0 \rightarrow X}(\omega) \\ \sigma_{tot}^{\gamma K^+ \rightarrow X}(\omega) &\equiv \sigma_{tot}^{\gamma K^- \rightarrow X}(\omega),\end{aligned}\tag{25}$$

following from C invariance of the electromagnetic interactions, are taken into account, one can see the sum rule (24), as well as all other possible sum rules obtained by combinations $K^+ \bar{K}^0, K^- K^0$, to be contained already in (23).

The last possibility is a consideration of a couple of mesons taken from the isomultiplet of pions and the isomultiplet of kaons leading to the following sum rules

$$\frac{1}{6}\pi^2\alpha[(\pm 1)\langle r_{\pi^\pm}^2 \rangle - (\pm 1)\langle r_{K^\pm}^2 \rangle] = \text{int}_{\omega_{th}}^\infty \frac{d\omega}{\omega} \left[\sigma_{tot}^{\gamma \pi^\pm \rightarrow X}(\omega) - \sigma_{tot}^{\gamma K^\pm \rightarrow X}(\omega) \right]\tag{26}$$

$$\frac{1}{6}\pi^2\alpha(\pm 1)\langle r_{\pi^\pm}^2 \rangle = \int_{\omega_{th}}^\infty \frac{d\omega}{\omega} \left[\sigma_{tot}^{\gamma \pi^\pm \rightarrow X}(\omega) - \sigma_{tot}^{\gamma K^0 \rightarrow X}(\omega) \right].\tag{27}$$

Now taking the experimental values [6]

$$(\pm 1)\langle r_{\pi^\pm}^2 \rangle = +0.4516 \pm 0.0108 \quad [fm^2] \quad (\pm 1)\langle r_{K^\pm}^2 \rangle = +0.3136 \pm 0.0347 \quad [fm^2]$$

one comes to the conclusion that in average

$$\begin{aligned}[\sigma_{tot}^{\gamma \pi^\pm \rightarrow X}(\omega) - \sigma_{tot}^{\gamma K^\pm \rightarrow X}(\omega)] &> 0 \\ [\sigma_{tot}^{\gamma K^- \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \bar{K}^0 \rightarrow X}(\omega)] &> 0,\end{aligned}\tag{28}$$

from where the following chain of inequalities for finite values of ω in average follow

$$\sigma_{tot}^{\gamma \pi^\pm \rightarrow X}(\omega) > \sigma_{tot}^{\gamma K^\pm \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \bar{K}^0 \rightarrow X}(\omega) > 0.\tag{29}$$

Subtracting up (23) or (24) from the relation (27), the sum rule (26) is obtained, what demonstrates a mutual consistency of all considered sum rules.

They have been derived in analogy with a derivation of the sum rule for a difference of proton and neutron total photo-production cross-sections [7], which is fulfilled with a very high precision. Therefore we believe that also the sum rules for total cross-sections of hadron photo-production on pseudoscalar mesons presented in this paper are correct.

VI. UNIVERSAL SUM RULE FOR TOTAL HADRON PHOTO-PRODUCTION CROSS-SECTIONS ON BARYONS

Now employing the Weicsäcker-Williams relation for baryons, taking a derivative according to \mathbf{q}^2 of both sides in q^2 -dependent baryon sum rule for $\mathbf{q}^2 \rightarrow 0$ and using the laboratory reference frame by $s_1 = 2m_B\omega$, one comes to the new universal baryon sum rule [4]

$$\begin{aligned} & \frac{1}{3}[F_{1B}(0)\langle r_{1B}^2 \rangle - F_{1B'}(0)\langle r_{1B'}^2 \rangle] - [\frac{\kappa_B^2}{4m_B^2} - \frac{\kappa_{B'}^2}{4m_{B'}^2}] = \\ & = \frac{2}{\pi^2\alpha} \int_{\omega_B}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma B \rightarrow X}(\omega) - \sigma_{tot}^{\gamma B' \rightarrow X}(\omega)] \end{aligned} \quad (30)$$

relating Dirac baryon mean square radii $\langle r_{1B}^2 \rangle$ and baryon anomalous magnetic moments κ_B to the convergent integral, in which a mutual cancellation of the rise of the corresponding total cross-sections for $\omega \rightarrow \infty$ is achieved.

VII. APPLICATION TO VARIOUS COUPLES OF OCTET BARYONS

According to the SU(3) classification of existing hadrons - there are known the following members of the ground state $1/2^+$ baryon octet (p , n , Λ^0 , Σ^+ , Σ^0 , Σ^- , Ξ^0 , Ξ^-). As a result, by using the universal expression (30) one can write down $8!/(2!(8-2)!) = 28$ different sum rules for total cross-sections of hadron photo-production on ground state $1/2^+$ octet baryons.

The most interesting from the point of view of experimental verification is proton-neutron sum rule [7]

$$\frac{1}{3}\langle r_{1p}^2 \rangle - \frac{\kappa_p^2}{4m_p^2} + \frac{\kappa_n^2}{4m_n^2} = \frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma n \rightarrow X}(\omega)]. \quad (31)$$

If one considers couples of the iso-triplet of Σ -hyperons and separately couples of the iso-doublet of Ξ -hyperons, one finds

$$\begin{aligned} & \frac{1}{3}[\langle r_{1\Sigma^+}^2 \rangle - [\frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} - \frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2}]] = \\ & = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)], \end{aligned} \quad (32)$$

$$\begin{aligned} & \frac{1}{3}[\langle r_{1\Sigma^+}^2 \rangle - \langle r_{1\Sigma^-}^2 \rangle] - [\frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} - \frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2}] = \\ & = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)], \end{aligned} \quad (33)$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Sigma^-}^2 \rangle - \left[\frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2} - \frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Sigma^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)],
\end{aligned} \tag{34}$$

and

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Xi^-}^2 \rangle - \left[\frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Xi^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)],
\end{aligned} \tag{35}$$

respectively, which represent the second class of the baryon sum rules.

The third class of the 23 baryon sum rules is found by a consideration of a couple of baryons always taken from different isomultiplets of the ground state $1/2^+$ baryon octet and take forms as follows

$$\begin{aligned}
& \frac{1}{3} \langle r_{1p}^2 \rangle - \left[\frac{\kappa_p^2}{4m_p^2} - \frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega)],
\end{aligned} \tag{36}$$

$$\begin{aligned}
& \frac{1}{3} [\langle r_{1p}^2 \rangle - \langle r_{1\Sigma^+}^2 \rangle] - \left[\frac{\kappa_p^2}{4m_p^2} - \frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega)],
\end{aligned} \tag{37}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1p}^2 \rangle - \left[\frac{\kappa_p^2}{4m_p^2} - \frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)],
\end{aligned} \tag{38}$$

$$\begin{aligned}
& \frac{1}{3} [\langle r_{1p}^2 \rangle + \langle r_{1\Sigma^-}^2 \rangle] - \left[\frac{\kappa_p^2}{4m_p^2} - \frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)],
\end{aligned} \tag{39}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1p}^2 \rangle - \left[\frac{\kappa_p^2}{4m_p^2} - \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)],
\end{aligned} \tag{40}$$

$$\begin{aligned}
& \frac{1}{3} [\langle r_{1p}^2 \rangle + \langle r_{1\Xi^-}^2 \rangle] - \left[\frac{\kappa_p^2}{4m_p^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)],
\end{aligned} \tag{41}$$

$$\begin{aligned}
& - \left[\frac{\kappa_n^2}{4m_n^2} - \frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega)],
\end{aligned} \tag{42}$$

$$\begin{aligned}
& - \frac{1}{3} \langle r_{1\Sigma^+}^2 \rangle - \left[\frac{\kappa_n^2}{4m_n^2} - \frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega)],
\end{aligned} \tag{43}$$

$$\begin{aligned}
& - \left[\frac{\kappa_n^2}{4m_n^2} - \frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)],
\end{aligned} \tag{44}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Sigma^-}^2 \rangle - \left[\frac{\kappa_n^2}{4m_n^2} - \frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)],
\end{aligned} \tag{45}$$

$$\begin{aligned}
& - \left[\frac{\kappa_n^2}{4m_n^2} - \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)],
\end{aligned} \tag{46}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Xi^-}^2 \rangle - \left[\frac{\kappa_n^2}{4m_n^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)],
\end{aligned} \tag{47}$$

$$\begin{aligned}
& -\frac{1}{3} \langle r_{1\Sigma^+}^2 \rangle - \left[\frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} - \frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega)],
\end{aligned} \tag{48}$$

$$\begin{aligned}
& -\left[\frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} - \frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)],
\end{aligned} \tag{49}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Sigma^-}^2 \rangle - \left[\frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} - \frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)],
\end{aligned} \tag{50}$$

$$\begin{aligned}
& -\left[\frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} - \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)],
\end{aligned} \tag{51}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Xi^-}^2 \rangle - \left[\frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)],
\end{aligned} \tag{52}$$

$$\begin{aligned}
& \frac{1}{3} \langle r_{1\Sigma^+}^2 \rangle - \left[\frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} - \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2} \right] = \\
& = \frac{2}{\pi^2 \alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)],
\end{aligned} \tag{53}$$

$$\begin{aligned}
& \frac{1}{3}[\langle r_{1\Sigma^+}^2 \rangle + \langle r_{1\Xi^-}^2 \rangle] - [\frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2}] = \\
& = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma\Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma\Xi^- \rightarrow X}(\omega)], \tag{54}
\end{aligned}$$

$$\begin{aligned}
& -[\frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2} - \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2}] = \\
& = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma\Sigma^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma\Xi^0 \rightarrow X}(\omega)], \tag{55}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3}\langle r_{1\Xi^-}^2 \rangle - [\frac{\kappa_{\Sigma^0}^2}{4m_{\Sigma^0}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2}] = \\
& = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma\Sigma^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma\Xi^- \rightarrow X}(\omega)], \tag{56}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3}\langle r_{1\Sigma^-}^2 \rangle - [\frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} - \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2}] = \\
& = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^-}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma\Sigma^- \rightarrow X}(\omega) - \sigma_{tot}^{\gamma\Xi^0 \rightarrow X}(\omega)], \tag{57}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3}[-\langle r_{1\Sigma^-}^2 \rangle + \langle r_{1\Xi^-}^2 \rangle] - [\frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2}] = \\
& = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^-}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma\Sigma^- \rightarrow X}(\omega) - \sigma_{tot}^{\gamma\Xi^- \rightarrow X}(\omega)]. \tag{58}
\end{aligned}$$

In order to evaluate the left hand sides of the derived sum rules and to draw out some phenomenological consequences, one needs the reliable values of Dirac baryon mean square radii $\langle r_{1B}^2 \rangle$ and baryon anomalous magnetic moments κ_B .

The latter are known (besides Σ^0 , which is found from the well known relation $\kappa_{\Sigma^+} + \kappa_{\Sigma^-} = 2\kappa_{\Sigma^0}$) experimentally (see the third column in Table I), however, to calculate $\langle r_{1B}^2 \rangle$ by means of the difference of the baryon electric mean square radius $\langle r_{EB}^2 \rangle$ and Foldy term, well known for all ground state octet baryons from the experimental information on the magnetic moments given by Review of Particle Physics [6]

$$\langle r_{1B}^2 \rangle = \langle r_{EB}^2 \rangle - \frac{3\kappa_B}{2m_B^2}, \tag{59}$$

TABLE I:

| B | $I_B[mb]$ | $m_B[GeV]$ | $\kappa_B[\mu_N]$ | $\langle r_{EB}^2 \rangle [fm^2]$ | $3\kappa_B/2m_B^2 [fm^2]$ | $\langle r_{1B}^2 \rangle [fm^2]$ |
|-------------|-----------|------------|-------------------|-----------------------------------|---------------------------|-----------------------------------|
| p | 0.9125 | 0.93827 | 1.7928 | 0.717 | 0.119 | 0.598 |
| n | 0.9100 | 0.93957 | -1.9130 | -0.113 | -0.127 | -0.240 |
| Λ^0 | 0.6454 | 1.11568 | -0.6130 | 0.110 | -0.029 | 0.081 |
| Σ^+ | 0.5679 | 1.18937 | 1.4580 | 0.600 | 0.060 | 0.660 |
| Σ^0 | 0.5648 | 1.19264 | 0.6490 | -0.030 | 0.027 | -0.003 |
| Σ^- | 0.5602 | 1.19745 | -0.1600 | 0.670 | -0.007 | 0.663 |
| Ξ^0 | 0.4647 | 1.31483 | -1.2500 | 0.130 | -0.042 | 0.088 |
| Ξ^- | 0.4601 | 1.32131 | 0.3493 | 0.490 | 0.012 | 0.502 |

we are in need of the reliable values of $\langle r_{EB}^2 \rangle$.

They are known experimentally only for the proton, neutron and Σ^- -hyperon.

Fortunately there are recent results [8] to fourth order in relativistic baryon chiral perturbation theory (giving predictions for the Σ^- charge radius and the Λ - Σ^0 transition moment in excellent agreement with the available experimental information), which solve our problem completely.

All necessary information is collected in Table I, where also numerical values of corresponding $\langle r_{1B}^2 \rangle$ are presented.

Calculating the left-hand side of all sum rules one finds

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma n \rightarrow X}(\omega)] = 2.0415mb, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) > \sigma_{tot}^{\gamma n \rightarrow X}(\omega) \quad (60)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)] = 2.0825mb, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) \quad (61)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)] = 4.2654mb, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) \quad (62)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)] = 2.1829mb, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) \quad (63)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Xi^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = 1.5921mb, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega) \quad (64)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega)] = 1.6673\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) \quad (65)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega)] = -0.4158\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) \quad (66)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)] = 1.6667\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) \quad (67)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)] = 3.8496\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) \quad (68)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)] = 1.7259\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) \quad (69)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma p \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = 3.3180\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma p \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega) \quad (70)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega)] = -0.3260\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma n \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) \quad (71)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega)] = -2.4573\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma n \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) \quad (72)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)] = -0.3747\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma n \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) \quad (73)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)] = 1.8082\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma n \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) \quad (74)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)] = -0.3156\text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma n \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) \quad (75)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma n \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = 1.2766 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma n \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega) \quad (76)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega)] = -2.0831 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) \quad (77)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega)] = -0.0006 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) \approx \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) \quad (78)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega)] = 2.1823 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) \quad (79)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)] = 0.0586 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) \quad (80)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Lambda^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = 2.1823 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Lambda^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega) \quad (81)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)] = 2.1417 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) \quad (82)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = 3.7338 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^+ \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega) \quad (83)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)] = 0.1168 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) \quad (84)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^0}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = 1.5732 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^0 \rightarrow X}(\omega) > \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega) \quad (85)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^-}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega)] = -2.1238 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Xi^0 \rightarrow X}(\omega) \quad (86)$$

$$\frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^-}}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) - \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega)] = -0.5316 \text{mb}, \quad \Rightarrow \quad \sigma_{tot}^{\gamma \Sigma^- \rightarrow X}(\omega) < \sigma_{tot}^{\gamma \Xi^- \rightarrow X}(\omega), \quad (87)$$

from where one gets the following chain of inequalities

$$\begin{aligned}\sigma_{tot}^{\gamma\Sigma^+\rightarrow X}(\omega) &> \sigma_{tot}^{\gamma p\rightarrow X}(\omega) > \sigma_{tot}^{\gamma\Lambda^0\rightarrow X}(\omega) \approx \sigma_{tot}^{\gamma\Sigma^0\rightarrow X}(\omega) > \\ \sigma_{tot}^{\gamma\Xi^0\rightarrow X}(\omega) &> \sigma_{tot}^{\gamma n\rightarrow X}(\omega) > \sigma_{tot}^{\gamma\Xi^-\rightarrow X}(\omega) > \sigma_{tot}^{\gamma\Sigma^-\rightarrow X}(\omega)\end{aligned}$$

for total cross-sections of hadron photo-production on ground state $1/2^+$ octet baryons to be valid in average for finite values of ω .

VIII. CONCLUSIONS

Considering the very high energy peripheral electron-hadron scattering with a production of a hadronic state X moving closely to the direction of initial hadron, then exploiting analytic properties of the forward retarded Compton scattering amplitude on the same hadron, for the case of small transferred momenta, new meson and baryon sum rules were derived. Evaluating the left-hand sides of the sum rules, the chains of inequalities for total cross-sections of hadron photo-production on pseudoscalar mesons and $1/2^+$ octet baryons have been found.

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